

# Representing Homology Classes by Locally Flat Surfaces of Minimum Genus\*

Ronnie Lee and Dariusz M. Wilczyński  
 YALE UNIVERSITY  
 UTAH STATE UNIVERSITY

## 1 Introduction

A necessary and sufficient condition will be given for a nontrivial homology class of a simply connected 4-manifold to be represented by a simple, topologically locally flat embedding of a compact Riemann surface.

## 2 Splittings of Hermitian Modules

We begin with an algebraic result.

**Theorem 1.** *The following is a commutative diagram of pointed hermitian modules.*

$$\begin{array}{ccccc}
 (M, h, z) & \xrightarrow{\pi_1} & (M_1, h_1, 0) & & \\
 \downarrow \pi_0 & \searrow \alpha & \downarrow \pi_{1d} & \searrow \alpha_1 & \\
 & \cong & (M'_1, h'_1, 0) \oplus H(\Lambda_1^k) & & \\
 & & \downarrow \pi_0 & \xrightarrow{\pi_1} & \\
 (M_0, h_0, z_0) & \xrightarrow{\pi_{0d}} & (M_d, h_d, 0) & & \\
 \downarrow \pi_0 & \searrow \alpha_0 & \downarrow \pi_{1d} & \searrow \alpha_d & \\
 & \cong & (M'_d, h'_d, 0) \oplus H(\Lambda_d^k) & & \\
 & & \downarrow \pi_0 & \xrightarrow{\pi_{0d}} & \\
 (M_0, h_0, z_0) & \xrightarrow{\pi_{0d}} & (M_d, h_d, 0) & & \\
 \downarrow \beta_0 & \searrow \beta_0 & \downarrow \beta_d & \searrow \beta_d & \\
 & \cong & (L, \lambda, x) \oplus H(\Lambda_0^k) & \xrightarrow{\pi_{0d}} & (L_d, \lambda_d, 0) \oplus H(\Lambda_d^k) \\
 & & \downarrow \beta_0 & & \downarrow \beta_d \\
 & & & & \cong \beta'_d \oplus id
 \end{array}$$

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